The first set of questions are really just questions to familiarize you regarding the background maths needed. Work through those if you need to and double check with the answers given to make sure you understand them. These will not be covered in the tutorials.

1. Probability. Lois knows that on average radio station RANDOM-FM plays 1 out of 4 of her requests. If she makes 3 requests, what is the probability that at least one request is played?

2. Let \( A \) and \( \mathbf{v} \) be defined as

\[
A = \begin{pmatrix}
1 & 1 & 3 \\
1 & 1 & -3 \\
3 & -3 & -3
\end{pmatrix} \quad \mathbf{v} = \begin{pmatrix}
1 \\
-1 \\
-2
\end{pmatrix}
\]

Calculate \( Av \). Is \( \mathbf{v} \) an eigenvector of \( A \), and if so what is the corresponding eigenvalue?

3. A random vector \( \mathbf{x} \) has zero mean and a diagonal covariance

\[
E(\mathbf{x}\mathbf{x}^T) = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

where \( E \) stands for expectation (or mean average) of a random variable. If \( \mathbf{y} = A^T \mathbf{x} \) (using \( A \) from Q2) what is the covariance of the resulting random vector \( \mathbf{y} \): \( E(\mathbf{y}\mathbf{y}^T) \)? You may use the fact that expectation is linear: \( E(R\mathbf{x}\mathbf{x}^T S) = RE(\mathbf{x}\mathbf{x}^T)S \). This shows how covariances change under linear transformations.

4. Find the partial derivatives of the function \( f(x, y, z) = (x + 2y)^2 \sin(xy) \).

5. Let \( \mathbf{x} \) be a Gaussian random variable with mean \( \mu \) and variance \( \sigma^2 \). What is the expected value of \( 2\mathbf{x}^2 \). Show what form the distribution of \( 2(\mathbf{x} - \mu)^2 \) takes.